AN AUTOMATIC HORIZONTAL CURVE RADII MEASUREMENT METHOD FOR ROADWAY SAFETY ANALYSIS USING GPS DATA

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ABSTRACT

Horizontal curves play a critical role in roadway driving safety by providing a smooth transition between two tangent sections. As one of the most fundamental elements in roadway geometry design, state departments of transportation (DOTs) need to measure the radii of horizontal curves to support their network-level safety analyses. However, the current practice for horizontal curve radii measurement, i.e. plan sheet reading, chord offset, etc., are time-consuming, labor-intensity and inaccurate. Even though some computer-aided radii measurement methods have been proposed using global positioning system (GPS) data and/or geographic information system (GIS) functions in recent years, these methods are not practical for a network-level analysis due to the lack of continuous curve radii measurement capability. This study, sponsored by the United States Department of Transportation Research Innovative Technology Administration (USDOT RITA) program, is aimed to propose a new automatic method using GPS data that can accurately and continuously conduct network-level curve radii measurement. The contributions of this paper include the following: 1) proposing an iterative circular fitting algorithm to accurately and continuously measure curve radii; 2) critically evaluating the accuracy of the proposed method using both simulation test and field test; 3) conducting a case study to demonstrate the feasibility of applying the proposed method in a network-level safety analysis. Results from both of the simulation test and the field test achieve a desirable accuracy, i.e. relative error of 1.48%±1.2% using synthetic data, and relative difference of 4.83%±1.8% comparing with the chord offset method. A case study on Interstate 285 shows promises of integrating the proposed method for achieving a reliable, efficient, network-level safety analysis. The proposed method can produce accurate and continuous curve radii measurement results efficiently, and lays a solid foundation for network-level safety analyses that rely on these measurements.

INTRODUCTION

Horizontal curves are one of the most important elements in roadway geometry design. The main function of a horizontal curve is to provide smooth transition between two tangent sections to ensure roadway driving consistency. Unfortunately, a disproportional number of serious vehicle crashes occur at horizontal curves despite the fact that curves only represent a fraction of the roadway network. As reported by the Federal Highway Administration (FHWA) Office of Safety, 27% of fatal crashes occurred at horizontal curves in the years of 2006-2008 (*1*). It is critical for state departments of transportation (DOTs) to focus on improving their horizontal curve safety. As the most fundamental step, horizontal curve radii measurement is essential for state DOTs to implement their horizontal curve safety programs.

Traditionally, state DOTs use plan reading method and/or chord offset method to determine a specific roadway curve radius. However, due to the nature of these methods, the process can be time-consuming and labor-intensive for field engineers. For example, plan reading method requires field engineer to go through excessive numbers of pages, yet it lacks of a direct correlation between the paper-based plan and the actual roadway environment. Chord offset method requires at least two field engineers to stretch a string with known length, and make multiple measurements on both inside and outside lane edges.

With the advancement in global positioning system (GPS)/geographic information system (GIS) technologies, GPS-based method becomes a feasible option for roadway curve radii measurement. The curve radius can be estimated from a measured deflection angle and the corresponding curve length information that can be acquired from GPS points (*2*). Several GIS applications have been developed to facilitate the angle and length measurement process by providing convenient curve digitization and measurement tools (*3; 4*). For a single curve, the existing GPS-based methods can achieve a fairly accurate measurement result (*2; 5*). However, to obtain a reasonable measurement results, the existing GPS-based method requires: 1) the GPS data be collected in the same lane, 2) the point of curve (PC) and the point of tangent (PT) be manually digitized for each curve. These requirements make the existing GPS-based methods unfeasible to conduct continuous measurement. It is impractical for state DOTs to adopt the existing GPS-based methods for a network-level safety analysis. Because many safety-related analyses rely on the availability of horizontal curve radii measurements (*6*), there is an urgent need for an accurate and continuous horizontal curve measurement method from state DOTs.

This paper proposes a new horizontal curve radii measurement method using GPS data that is aimed at state DOTs' need for network-level analyses. The contribution of this paper is in threefold: 1) developing an iterative circular fitting to robustly and continuously measure curve radii; 2) designing and conducting both simulation test and field test to comprehensively evaluate the performance of the proposed method; 3) conducting a case study to demonstrate the capacity of the proposed method in a network-level safety analysis using the actual data collected on Interstate 285 (I-285) in Atlanta, Georgia.

This paper is organized as follows. The first section identifies the research need and objective. The second section presents the proposed method with a focus on the new iterative circular fitting algorithm. The third section presents the experimental test conducted to evaluate the performance of the proposed method using both synthetic data and field data. The fourth section uses a case study to demonstrate the capability of the proposed method supporting a network-level safety analysis. The fifth section summarizes the conclusions and recommendations of this study.

PROPOSED METHOD

In this section, the proposed horizontal curve radii measurement method is presented. This method contains four primary steps as shown in FIGURE 1, including GPS post-processing, iterative circular fitting, circular fitting validation and tangent section identification.

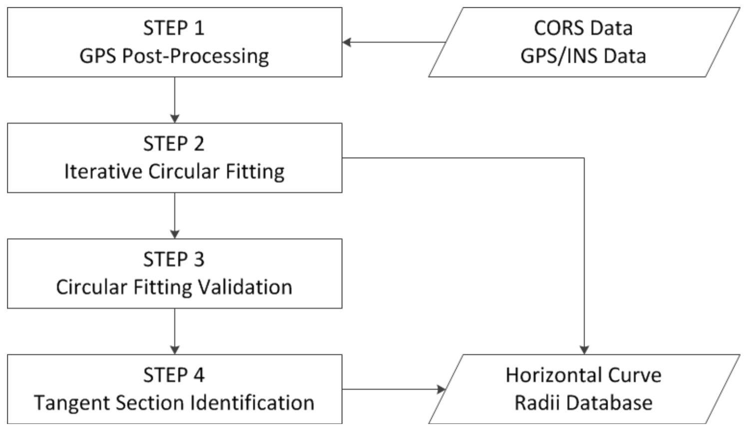


FIGURE 1 Flowchart for the proposed curve radii measurement method

STEP 1 - GPS Post-Processing

The objective of this step is to adjust the raw GPS data by triangulating the data from the continuous operating reference stations (CORS). Raw GPS data is acquired from the data collection in real time, e.g. 1Hz. Because the data is collected through a mobile platform, many factors will impact the accuracy of the raw GPS data, e.g. the dilution of precision (DOP), temporary satellite loss, limited dwell time, etc. The triangulation using CORS is to reduce the impact of these negative factors on the accuracy. Inertial navigation system (INS) data is also used in this step to compensate the loss of GPS data at the locations where satellites are not visible. This step can be achieved using commercially available software.

STEP 2 - Iterative Circular Fitting

The objective of this step is to automatically identify the delineation of the roadway using the post-processed GPS points, i.e. straight section, curved section and tangent section, and to conduct circular fitting to obtain the radii for each curve. The iterative process is proposed to ensure that an optimized number of GPS points are grouped for each horizontal curve, while the regression-based circular fitting process is introduced to improve the robustness of the measurement to the disturbances in GPS points.

In previous studies, automatic circular fitting methods have been proposed with accurate results (*7-9*). However, it is not feasible to directly introduce these methods for horizontal curve radii measurement without identifying the delineation of the roadway. These circular fitting methods only take a fixed number of points for circular fitting. Hence, no matter how robust the circular fitting method is, the fitting results are severely biased by the selected number of points. In this paper, an iterative circular fitting algorithm is proposed to the GPS points to obtain the initial circular fitting results. Instead of selecting a fixed number of neighboring points for fitting, at each GPS point, different numbers of neighboring points are attempted until arriving at the least fitting error. The number resulted in the least fitting error will be associated with this group of GPS points. The fitting error is measured by the fitness of the actual GPS points to the fitted circle. Once the number of neighboring points is selected for the current group of GPS points, the next circular fitting will be started by skipping the current group of points. Hence, for each group of GPS points, the optimized numbers of neighboring GPS points are selected through iterations. FIGURE 2 shows the pseudo code for the iterative circular fitting.

PSEUDO CODE 1 - ITERATIVE CIRCULAR FITTING

Input a GPS sequence (x, y)

Initialize the piece struct $L(\text{index}, \text{interval}, x_{center}, y_{center}, \text{Curvature}, \text{flag})$

for each GPS point (x_i, y_i)

  for each interval j (i.e. number of neighbors)

    Compute circle center (a_{ij}, b_{ij}) and radius R_{ij} using Kasa circle fitting

    Compute error term $E_j = \frac{1}{j} \cdot \sum_{k=i}^{i+j} \left(\sqrt{\left(\left(x_k - a_{ij}\right)^2 + \left(y_k - b_{ij}\right)^2 \right)} - R_{ij} \right)$

  end

  $m = \text{argmin}_j \left(E_j\right)$

  Add record of $L_N = L(i, m, a_{im}, b_{im}, {R_{im}}^{-1})$ to L

  $N = N + 1$

  $i = i + m + 1$

end

FIGURE 2 Pseudo code for iterative circular fitting algorithm

There are two questions remained for this algorithm: 1) selection of the number of neighboring points for iteration; 2) selection of the circular fitting method.

- The principle of selecting neighboring points for circular fitting is in twofold. On one hand, the number of points should be sufficient to minimize the impact of local GPS disturbance on the circular fitting. On the other hand, the number of points should only be necessary to minimize the impact of the GPS points that are associated with the tangent sections. In this paper, fifty iterations were used for the iterative circular fitting (e.g. 10 to 500 points with an increment of 10).
- The principle of selecting the circular fitting algorithm is also in twofold. On one hand, the method should be robust to the noise. On the other hand, the method should be computationally efficient, so that the iterative circular fitting algorithm can be practically implemented. In this paper, the classic circular fitting method proposed by Kasa (*7*) is selected for its computational efficiency and robustness to noise.

STEP 3 – Circular Fitting Validation

The initial circular fitting results cannot be used directly as the curve radii, because the iterative circular fitting algorithm computes the curve radii purely based on the data term from the selected number of neighboring points. Therefore, the validity of the initial curve radii should be checked first. Based on the nature of circular fitting, the straight section and the curved section can be confidently validated. FIGURE 3 shows the pseudo code for the circular fitting validation. It should be noted that curvature is used here to avoid infinity values in radius.

- For straight sections, the error of the circular fitting will not be reduced by increasing the number of neighboring GPS points for circular fitting. Therefore, the group of GPS points representing a straight section typically contains less GPS points and results in a large radius. By constraining these two parameters, i.e. BoundA and BoundB, the straight sections can be validated. BoundA and BoundB are heuristic parameters that can be defined and adjusted by the users. In this study, BoundA=50 and BoundB=0.00001 are used.
- For curved sections, the error of the fitting will be reduced by increasing the number of GPS points until the selection of GPS points start to include tangent sections. Therefore, the group of GPS points representing a curved section typically contains more GPS points and results in a relatively small radius. By constraining these two parameters, i.e. BoundC and BoundD, the curved sections can be validated. BoundC and BoundD are heuristic parameters that can be defined and adjusted by the users. In this study, BoundC=250 and BoundD=0.00002 are used.

```
PSEUDO CODE 2 - CIRCULAR FITTING VALIDATION
For each record in L
    if L_i.interval < BoundA OR L_i.Curvature < BoundB
        L_i.flag = "Straight Section"
        L_i.Curvature = 0
    end
    if L_i.interval > BoundC OR L_i.Curvature > BoundD
        L_i.flag = "Curved Section"
    end
end
```

FIGURE 3 Pseudo code for circular fitting validation algorithm

STEP 4 – Tangent Section Identification

After the circular fitting validation, only the tangent sections remain undefined. By connecting the validated sections for straight and curved sections, the tangent sections can be defined. If the consecutive sections are both straight sections, the section in between should be straight section. If the consecutive sections include at least a curved section, the section in between should be tangent sections. The radius for each point of this section is computed by interpolating the radii values in the neighboring sections. FIGURE 4 shows the pseudo code for identifying tangent sections.

```
PSEUDO CODE 3 - IDENTIFICATION OF TANGENT SECTIONS
For each record in L where L_i.flag = NULL
    if L_{i-1}.flag = "Straight Section" AND L_{i+1}.flag = "Straight Section"
        L_i.flag = "Straight Section"
        L_i.Curvature = 0
    else
        L_i.flag = "Tengent Section"
        Interpolate each point within L_i using L_{i-1}.Curvature and L_{i+1}.Curvature
    end
end
```

FIGURE 4 Pseudo code for the identification of tangent sections

By validating the straight and curved sections in STEP 3, and identifying the tangent sections in STEP 4, the complete GPS data is delineated into different sections, within which the radii are calculated based on the optimized number of neighboring points for circular fitting. Following the above mentioned algorithm, the number of GPS points for circular fitting is adaptive, while the derived radii measurements are continuous and consistent. The derived horizontal curve radii are recorded into a geo-database, where each GPS point is associated with a radius measurement.

EXPERIMENTAL TEST

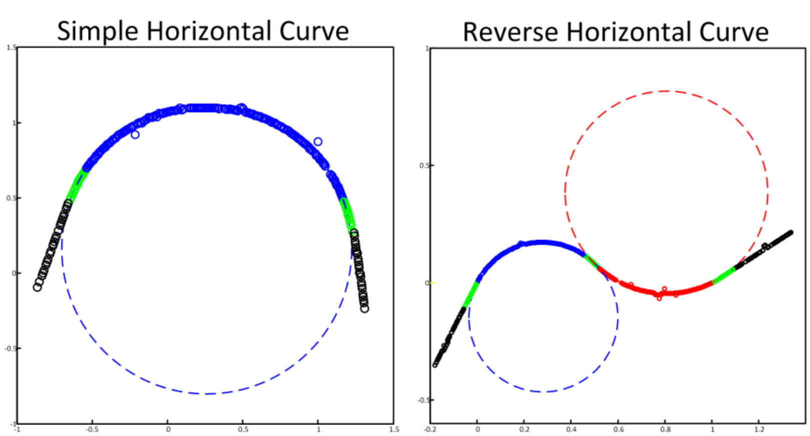
The objective of the experimental test is to comprehensively evaluate the performance of the proposed method. Simulation tests using synthetic curve data are performed first to evaluate the accuracy of the proposed method. Field tests using real-world curve data are then conducted to evaluate the performance of the proposed method by comparing with the current practice, i.e. chord offset method.

Simulation Test

The objective of the simulation test is to assess the accuracy of the proposed method using well-controlled dataset and known ground truth. In this simulation test, four commonly used types of horizontal curves for roadway geometry design are first tested (*10*), including:

- Simple curve: A simple curve is an arc of a circle;
- Reverse curve: A reverse curve consists of two simple curves joined together, but curving in opposite directions;
- Compound curve: A compound curve normally consists of two simple curves joined together, but curving in the same direction;
- Spiral curve: A spiral curve contains a varying radius.

In order to test the robustness of the proposed method, several types of GPS disturbances observed from the real GPS track are simulated in the synthetic data, including random GPS error, missing GPS points, local vehicle maneuvering. These GPS disturbances can locally interrupt the continuity of the GPS track. FIGURE 5 shows an example of the synthetic data and the fitted circles derived from the proposed method for four different types of horizontal curves. The red and blue sections represent the curved sections with corresponding fitted circles. The black sections represent the straight sections, while the green sections represent the tangent sections.



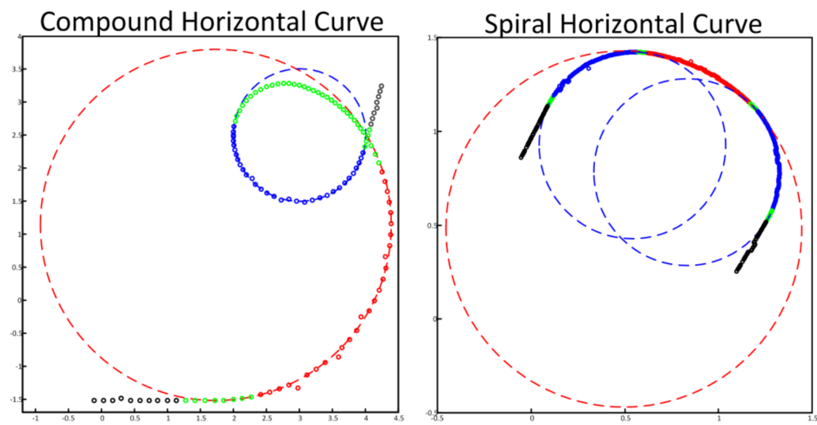


FIGURE 5 Synthetic curve data and the circular fitting results

To quantify the accuracy of the proposed method, a long synthetic data containing randomly generated horizontal curves is created. The data consists of a thousand horizontal curves and the ground truth for each curve is established when generating the curves. The average relative error ($\frac{1}{N} \cdot \sum_{i=1}^{N} \frac{|Ground\ truth_i - Measurement_i|}{Ground\ truth_i} \cdot 100\%,\ N = 1000$) of the radii measurement is 1.48%±1.2%, which is better than the GPS-based measurement method as presented by Carlson et al. (*2*) and much better than the developed GIS-based measurement methods as presented by Rasdorf et al. (*5*). The only case large error occurs in this test is for spiral horizontal curve, when the radii of the adjacent curves have small difference. The proposed method considers the spiral curve as a large simple curve, so that the center curve of the spiral is under-estimated while the two side curves in the spiral are over-estimated.

Field Test

The objective of field test is to evaluate the performance of the proposed method in the real world environment. Instead of establishing ground truth which is not possible in field, chord offset method was used to represent the current practice in state DOTs. FIGURE 6 shows data collection using the chord offset method, and the layout of the testing roads.

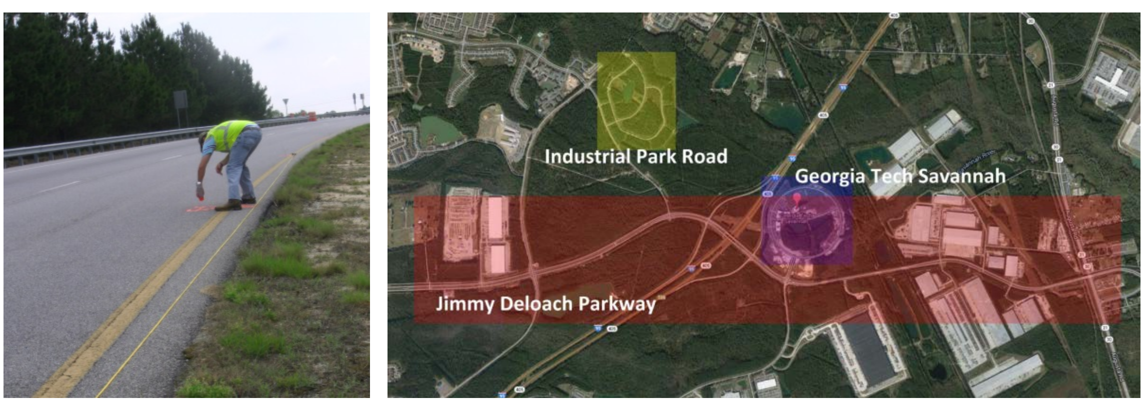


FIGURE 6 Data collection using chord offset and the layout of the testing roads

Three testing datasets were collected, including 25 simple curves with different radii. The corresponding GPS data was collected on these three roads. The proposed method is separately applied to the three testing datasets.

- Georgia Tech Savannah campus: Georgia Tech Savannah campus contains consistent curve radii on a circular loop road (i.e. five radii ranging between 958 ft. to 1073 ft.).
- Jimmy Deloach Parkway: Jimmy Deloach Parkway is a minor arterial with majority of large and smooth curves (i.e. ten radii ranging between 1366 ft. to 5682 ft.).
- Industrial Park Road: Industrial Park Road is a local road with majority of sharp curves (i.e. ten radii ranging between 233 ft. to 834 ft.).

TABLE 1 summarizes the comparison result between the proposed method and the chord offset method. It can be observed that the proposed method performs consistently well for different radii. The average relative error ($\frac{1}{N} \cdot \sum_{i=1}^{N} \frac{|Chord\ Offset_i - Proposed\ Method_i|}{Chord\ Offset_i} \cdot 100\%,\ N = 25$) of the radii measurement is 4.83%±1.8%, which is comparable the GPS-based measurement method as presented by Carlson et al. (*2*) and much better than the developed GIS-based measurement method as presented by Rasdorf, et al. (*5*). It is observed that for the locations with larger radii (e.g. Jimmy Deloach Parkway), the absolute difference between the chord offset method and the proposed method is slightly greater. This is because the chord offset method tends to produce a larger measurement error when the radii are large with a smaller offset to measure.

TABLE 1 Field Test Results on Horizontal Curve Radii Measurement

Site	Location	Chord Offset Method (ft.)	Proposed Method (ft.)	Relative Difference
C-1	Georgia Tech Savannah Campus	959	1001	4.43%
C-2	Georgia Tech Savannah Campus	1013	997	-1.53%
C-3	Georgia Tech Savannah Campus	1074	1002	-6.70%
C-4	Georgia Tech Savannah Campus	1001	986	-1.51%
C-5	Georgia Tech Savannah Campus	966	1006	4.19%
J-1	Jimmy Deloach Parkway	1894	1938	2.31%
J-2	Jimmy Deloach Parkway	1462	1389	-5.02%
J-3	Jimmy Deloach Parkway	1471	1407	-4.35%
J-4	Jimmy Deloach Parkway	1367	1419	3.84%
J-5	Jimmy Deloach Parkway	1454	1509	3.79%
J-6	Jimmy Deloach Parkway	3677	3861	5.01%
J-7	Jimmy Deloach Parkway	5208	5503	5.66%
J-8	Jimmy Deloach Parkway	3788	3980	5.07%
J-9	Jimmy Deloach Parkway	5682	5342	-5.98%
J-10	Jimmy Deloach Parkway	3600	3306	-8.17%
I-1	Industrial Park Road	437	470	7.56%
I-2	Industrial Park Road	478	447	-6.57%
I-3	Industrial Park Road	480	460	-4.21%
I-4	Industrial Park Road	580	565	-2.55%
I-5	Industrial Park Road	233	220	-5.63%
I-6	Industrial Park Road	243	262	7.62%
I-7	Industrial Park Road	253	241	-4.93%
I-8	Industrial Park Road	250	265	6.18%
I-9	Industrial Park Road	730	700	-4.07%
I-10	Industrial Park Road	834	801	-3.97%

CASE STUDY: AN INTERSTATE 285 SAFETY APPLICATION

Horizontal curve is one of the most important and fundamental elements in roadway geometry design. Not only the measured radii is critical information by itself for assessing the adequacy of the roadway design in a network level, but more importantly integrating the measured radii with other roadway geometry elements will provide a more meaningful safety-related product for state DOTs. This case study is presented to demonstrate the usefulness of the proposed method in analyzing the adequacy of roadway to avoid lane departure accident.

According to the policy on geometry design of highway and streets by American Association of State Highway and Transportation Officials (AASHTO) (*11*), the adequacy of the curve is assessed by computing the formula

$$I = \frac{e+f}{1-ef} \cdot (\frac{gR}{v^2})$$

where:

e = rate of roadway super-elevation, percent;
f = side friction (demand) factor;
v = vehicle speed, ft./s;
g = gravitational constant, 32.2 ft./s^2;
v = vehicle speed, mph;
R = radius of curve, ft.

The non-negative I is the index that describes the roadway departure risk for a curved section. If I is greater than one, the force provided by the side friction and super-elevation guarantees a sufficient centripetal force, so the curved segment can be assumed to be safe. When I ranges from zero to one, the curve is unsafe, with a high tendency of vehicle skidding and lane departure. The unsafe sections with I that is much less than one are of the most concern for further investigation.

The complete I-285 in Atlanta, Georgia, covering 63 miles, was collected for this case study. FIGURE 7 shows curve radii (R) along I-285 derived from the proposed method. By applying the super elevation measurement (e) derived from Tsai, et al. (*12*), driving speed (v) obtained from the Georgia Department of Transportation (GDOT), and the typical side friction factor (f) recommended by AASHTO, i.e. 0.12, the safety indices representing lane departure are computed for the entire I-285. FIGURE 8 shows the distributions of the safety indices that are smaller than 0.3 along I-285 that represent the most safety concern. Constrained by the information on the detailed pavement surface condition, i.e. side friction factor, the results shown in FIGURE 8 does not necessarily reflect the actual safety risk on I-285. However, the results are informative for state DOTs and clearly demonstrate the capability of the network-level analyzing capability. For example, engineers can use the spatial distribution of the safety index as shown in FIGURE 8 to identify the potential locations with the highest safety risk for a focused evaluation (e.g. a site visit). In addition, because all the measurements are geo-referenced, the corresponding GIS application can be developed to facilitate the safety-related task by directly importing and re-computing the derived measurements, creating what-if analysis, etc. It should be noted that the complete process from raw GPS data collection to the final result delivery as shown in FIGURE 8 only take approximately six hours (i.e. 10 mile/hour), which is efficient and practical for a large network-level analysis.

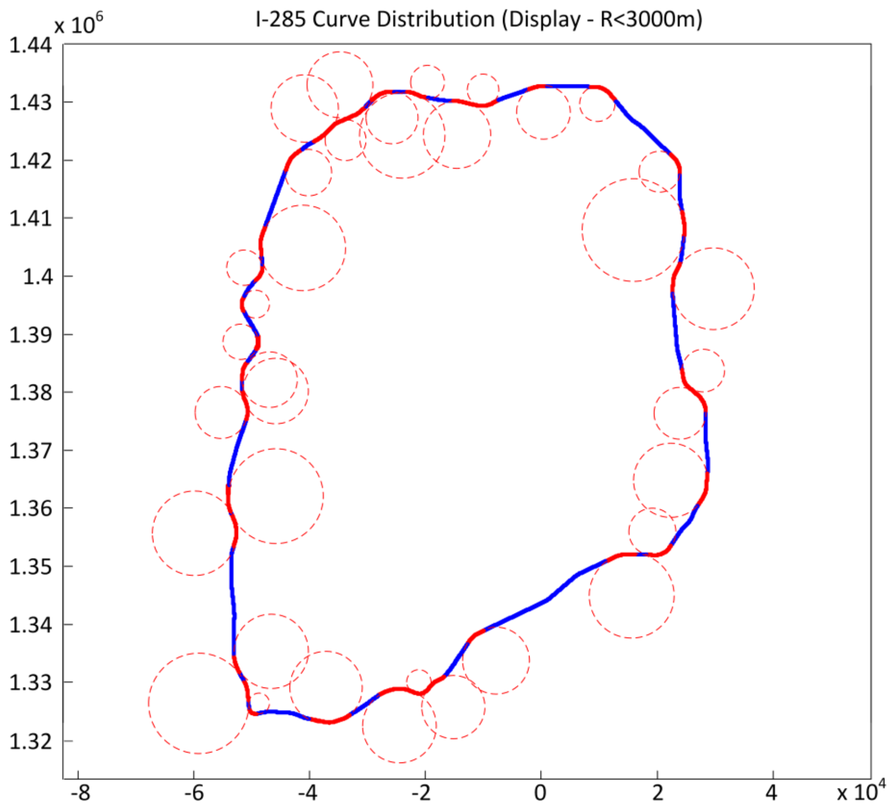


FIGURE 7 Spatial distribution of curves on I-285

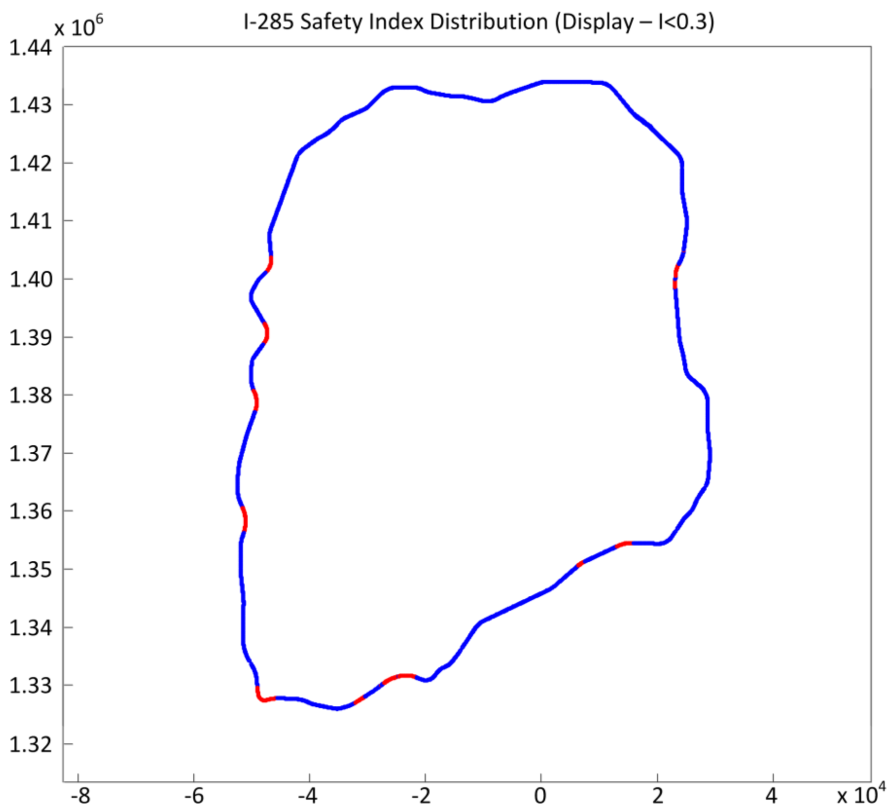


FIGURE 8 Spatial distribution of computed safety index on I-285

CONCLUSIONS AND RECOMMENDATIONS

Horizontal curve plays a critical role in providing smooth transition between two tangent sections to ensure roadway driving consistency. As one of the most fundamental elements in roadway geometry design, the radii of the horizontal curves need be accurately and continuously measured to enable state DOTs' network-level safety analysis. However, the current practice for curve radii measurement, i.e. plan sheet reading or chord length measurement, are time-consuming, labor-intensity and inaccurate. Although, some computer-aided methods using GPS data have been proposed for a single curve radius measurement, all of these methods require manual input for identifying the PC and PT of the curve for measurement, which disqualifies them to satisfy state DOTs' need for a network-level analysis. In this study, an automatic horizontal curve radii measurement method using an iterative circular fitting algorithm on GPS data, is proposed to address state DOTs' need. The performance of the proposed method is quantitatively evaluated using both simulation test and field test. The feasibility of applying the proposed method in a network-level safety analysis is also evaluated using a case study on I-285 in Atlanta, Georgia. The following points summarize the major findings of this study.

- From the simulation test, results using the synthetic data show that the proposed method can achieve desirable measurement accuracy, i.e. relative error 1.48%±1.2%, while operated continuously. The accuracy derived from the proposed method outperforms the existing GPS-based methods (*2; 5*) that can only measure a single curve at one time.
- From the field test, results from the real-world data show that the proposed method can achieve desirable measurement accuracy, i.e. relative difference from the chord offset method is 4.83%±1.8%. The accuracy derived from the proposed method delivers comparable or better performance comparing with the existing GPS-based methods (*2; 5*) that can only measure a single curve at one time.
- The case study on I-285 has demonstrated that the proposed method has the capability in conducting a network-level analysis efficiently (i.e. 10 mile/hour). The results derived from the proposed method can adequately support state DOTs' network-level safety analyses.

The following are the recommendation for future research:

- Additional field data with different curve types should be used for future performance evaluation of the proposed method.
- Further study should be undertaken to establish a systematic approach for determining the heuristic parameters used in the proposed method, so as to provide a reliable guideline for state DOTs to follow.
- Additional safety-related applications should be explored by taking advantage the derived continuous curve radii measurements from the proposed method.

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DISCLAIMER

The views, opinions, findings and conclusions reflected in this paper are the responsibility of the authors only and do not represent the official policy or position of the USDOT RITA, or any State or other entity.

REFERENCE

[1] FHWA. RDs and Curves, 2005-2008.In, FHWA Office of Safety, 2010.

[2] Carlson, P., M. Burris, K. Black, and E. Rose. Comparison of Radius-Estimating Techniques for Horizontal Curves. *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 1918, No. -1, 2005, pp. 76-83.

[3] Hans, Z., R. Souleyrette, and C. Bogenreif. Horizontal Curve Identification and Evaluation.In, 2012.

[4] Bonneson, J., J. M. M. Pratt, and P. Carlson. Horizontal Curve Signing Handbook.In, Texas Transportation Institute, 2007.

[5] Rasdorf, W. J., D. J. Findley, C. V. Zegeer, C. A. Sundstrom, and J. E. Hummer. Evaluation of GIS Applications for Horizontal Curve Data Collection. *Journal of Computing in Civil Engineering*, Vol. 26, No. 2, 2012, pp. 191-203.

[6] Torbic, D. J., D. W. Harwood, D. K. Gilmore, R. Pfefer, T. R. Neuman, K. L. Slack, and K. K. Hardy. Volume 7: A Guide for Reducing Collisions on Horizontal Curves.In *Guidance for Implementation of the AASHTO Strategic Highway Safety Plan, No. 7*, Transportation Research Board, 2004.

[7] Kasa, I. A circle fitting procedure and its error analysis. *Instrumentation and Measurement, IEEE Transactions on*, Vol. IM-25, No. 1, 1976, pp. 8-14.

[8] Pratt, V. Direct least-squares fitting of algebraic surfaces. *SIGGRAPH Comput. Graph.*, Vol. 21, No. 4, 1987, pp. 145-152.

[9] Taubin, G. Estimation of planar curves, surfaces, and nonplanar space curves defined by implicit equations with applications to edge and range image segmentation. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, Vol. 13, No. 11, 1991, pp. 1115-1138.

[10] Mansour, S. A. Horizontal, Spiral and Vertical Curves.In *Surveying for California Civil PE License*, 2010.

[11] AASHTO. *A Policy on Geometric Design of Highways and Streets-2011 AASHTO Green Book*. Washington D.C., AASHTO, 2011.

[12] Tsai, Y., C. Ai, Z. Wang, and E. Pitts. A Mobile Cross Slope Measurement Method Using LiDAR Technology. *92nd Transportation Research Board Annual Meeting*, 2012.